The lowest oscillation mode of a pendant drop

Jong Hoon Moon and Byung Ha Kang
School of Mechanical and Automotive Engineering, Kookmin University, Seoul 136-702, Korea

Ho-Young Kim[a)]
School of Mechanical and Aerospace Engineering, Seoul National University, Seoul 151-744, Korea

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The lowest oscillation mode of a pendant drop has long been conceived to be the longitudinal vibration, i.e., periodic elongation and contraction along the longitudinal direction. However, here we experimentally show that the rotation of the drop about the longitudinal axis is the oscillation mode of the lowest resonance frequency. This rotational mode can be invoked by periodic acoustic forcing and is analogous to the pendulum rotation, having the frequency independent of the drop density and surface tension but inversely proportional to the square root of the drop size. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2174027]

Liquid drops in contact with solid surfaces oscillate when exposed to external disturbances. Predicting the resonance frequency of the vibration of the pendant or sessile drops is of a great interest in such applications as drop ejection[1] and cleaning[2] processes, where drops vibrated at the adequate frequency can easily disengage from the substrate.[3] Also the pendant drop oscillation is used for dynamic surface tension measurement[4] as well as for the fundamental understanding of nonlinear dynamics.[5] Strani and Sabetta[6] theoretically analyzed the oscillation of a drop in contact with a concave solid support and showed that its first mode reduces to a zero-frequency rigid motion as the support size tends to zero. As shown in Fig. 1(b), the first mode oscillation of a pendant drop corresponds to the periodic elongation and contraction of the drop in the z direction. We refer to this first mode as the longitudinal mode henceforth. This longitudinal oscillation of a pendant or sessile drop was observed both by experiments[3,5,7] and numerical analyses.[8,9]

The studies on pendant or sessile drops have so far assumed that the lowest oscillation mode of a pendant drop is the longitudinal vibration without exception. However, here we make a novel experimental observation that there exists a vibration mode that occurs in a frequency that is lower than that of the longitudinal mode. Therefore, the aim of this Letter is to report the truly lowest oscillation mode of pendant drops.

Noting that drops only exhibit the first (longitudinal) or higher mode oscillation under mechanical[3,5,8,9] and electrical disturbance[7] in the longitudinal direction, we imposed acoustic forcing on a pendant drop so that the drop would freely choose its preferred direction of oscillation. When increasing the frequency of the acoustic wave from zero, the first resonant oscillation mode emerged, by which the pendant drop rotated about the z axis. The resonance frequency of this oscillation mode was substantially lower than that of the longitudinal mode. In the following, we describe the experimental methods and present a physical interpretation of the results.

In the experiment, a liquid drop pendant from a Teflon surface experienced the acoustic forcing generated by a woofer as illustrated in Fig. 2. The liquids used in this work were water and water-methanol mixtures, whose physical properties are listed in Table I. All the liquids have a nearly identical contact angle on the Teflon surface (111°) while their surface tension coefficients differ. Thus it is possible to study the effects of the material properties and the drop size on resonance frequencies while the contact angle is fixed. The volume of the drops used in this work ranged from 2 to 7 μl. The motion of the drop was recorded by a high-speed video system. In searching for the forcing frequency that excited the resonant response of the drop, we measured the temporal evolution of the lateral swing, Δs in Fig. 1(a), and the longitudinal height, Δl in Fig. 1(b), at different forcing frequencies. As Fig. 3 illustrates, we find sharp peaks corresponding to the resonance in the oscillations of Δs and Δl for each drop size.

The higher-frequency resonance mode corresponded to the longitudinal mode of a pendant drop as shown in Fig. 4(c). This frequency was identical to the dominant frequency of a freely vibrating pendant drop when an impulse in the longitudinal direction was given.[3] On the other hand, the resonance occurring at the lower frequency gives rise to a novel drop motion which is first reported here. In this resonance mode, referred to as the rotational mode, the drop rotates around the z axis as shown in Figs. 4(a) and 4(b). Identical rotational motion was also observed when the acoustic pressure was provided from the bottom as well as from the side as originally illustrated in Fig. 2. When viewed along the r axis, the drop appears to rock from side to side, exhibiting severe asymmetric deformation as shown in Fig. 4(a). However, when viewed along the z axis as in Fig. 4(b), the drop rotates about a point. This center of rotation, the intersection of the cross in Fig. 4(b), corresponds to the center of the circular contact area in Fig. 4(a). Therefore, this result shows that the lowest oscillation mode of a pendant drop corresponds to the rotation of the drop about the z axis.

[a)]Author to whom correspondence should be addressed. Electronic mail: hyk@snu.ac.kr

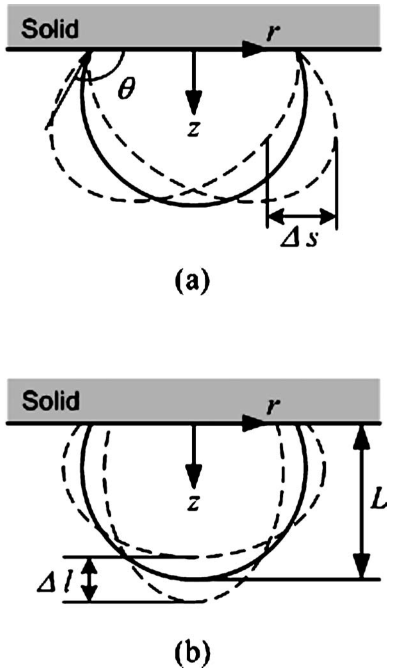

FIG. 1. Vibrating pendant drops. The solid line is the stationary drop and the dashed lines are the drop shapes undergoing (a) lateral and (b) longitudinal oscillations.

To compare this resonance frequency with the drop's natural frequency as was performed for the longitudinal vibration above, we gave the drop an impulse in the lateral direction. Although the pendant drop rocked from side to side instead of rotating under this lateral impact, its dominant frequency was consistent with the resonance frequency of the rotational mode. Here we note that the natural frequency of the pendant drop is identical to its resonance frequency when the viscous damping is negligible. To estimate the effects of viscosity, we scale the boundary layer thickness as $\delta \sim (\nu/f)^{1/2}$, where ν is the kinematic viscosity and f is the drop oscillation frequency. For δ to be comparable to the drop size in our experiments, the frequency should be on the order of unity, one order lower than the currently observed resonance frequencies of the rotational mode. We also find from the free vibration experiments that the drop oscillation mode is critically affected by the direction of external forcing as well as its frequency. Thus the acoustic means employed here is an ideal forcing method to find the lowest oscillation mode because its effect is nearly independent of orientation.

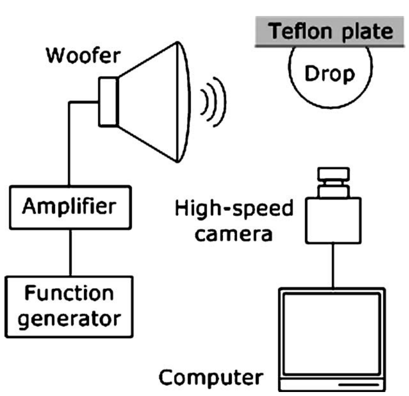

FIG. 2. A schematic of the experimental setup.

TABLE I. Physical properties of the liquids at 20 °C.

	Density (kg/m^2)	Surface (N/m)	Viscosity (mPa·s)
Water	998	0.073	0.890
Water-5 wt. % methanol	997	0.063	0.941
Water-10 wt. % methanol	995	0.059	1.051

Seeking the physical explanation of the rotational mode, we measured the resonance frequencies for various sizes of three different liquids of drops. Figure 5(a) shows the measured resonance frequencies versus drop size represented by R. The equivalent radius R is given by $R = L/(1-\cos\theta)$, where L is the longitudinal length and θ is the contact angle. We chose R as a representative drop size based on the result of Ref. 3, which showed that using R in Strani and Sabetta's model[6] predicted the natural frequency close to the experimental resonance frequency of the longitudinal mode.

In Fig. 5(a), it is first noted that the resonance frequencies of the rotational mode for different liquids are almost indiscernible while the resonance frequencies of the longitudinal mode strongly depend on liquid types. Second, by ana-

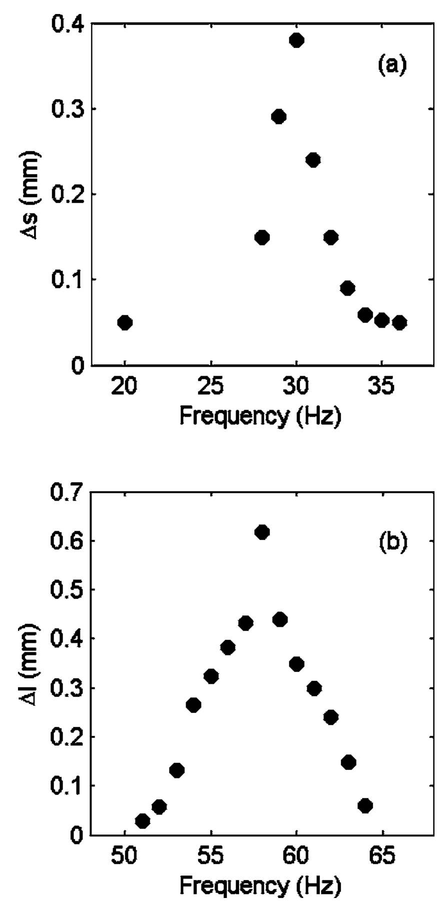

FIG. 3. Oscillation amplitude of the pendant water drop having the volume 7 μl under acoustic forcing vs frequency. (a) The swinging amplitude (Δs). (b) The longitudinal amplitude (Δl).

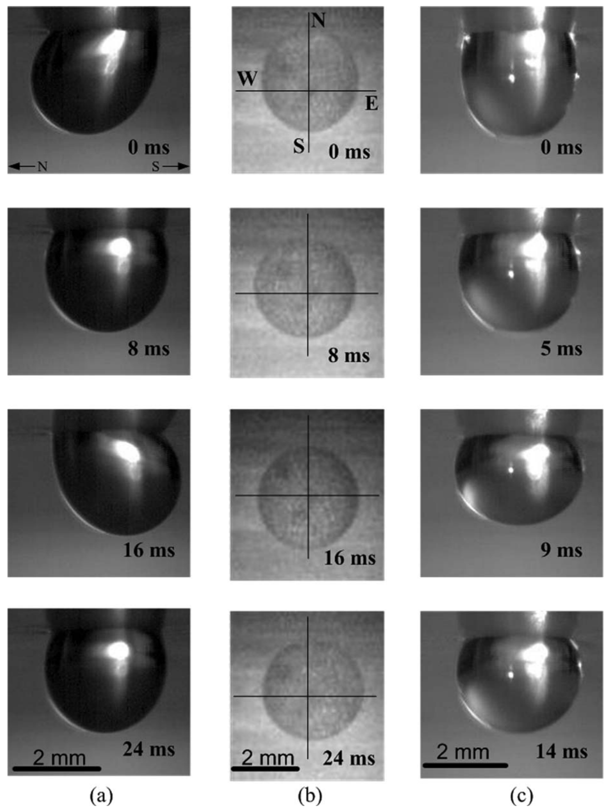

FIG. 4. Images of the resonant oscillations of the pendant water drops having the volume 7 μl. (a) and (b) show the rotational-mode oscillation, under the acoustic forcing of 30 Hz, viewed along the r and the z axes, respectively. In (b), the cross is stationary throughout time while the drop rotates counterclockwise, starting from the north (0 s) then moving to the west (8 ms), the south (16 ms), and the east (24 ms). It should be noted that the rotating circle corresponds to the equator of the drop in (a), not the contact line which does not move. (c) Longitudinal oscillation under the acoustic forcing of 58 Hz.

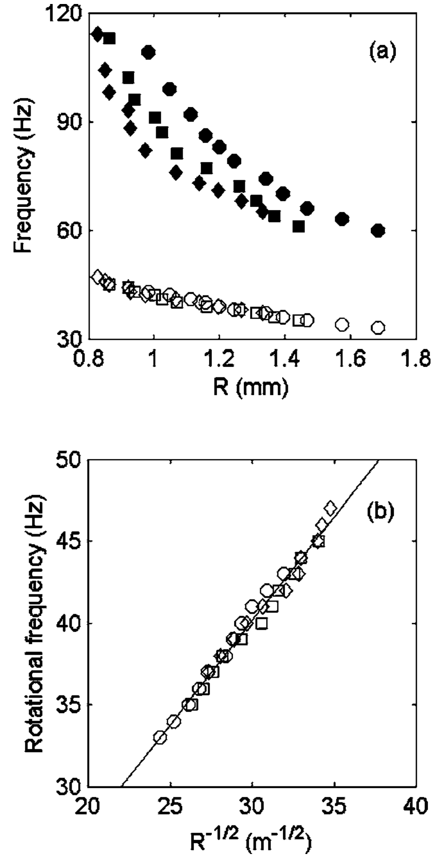

FIG. 5. (a) The measured resonance frequency for different sizes of drops: circles for water, squares for water-5 wt. % methanol, and diamonds for water-10 wt. % methanol. Open and filled symbols denote the rotational and the longitudinal modes, respectively. (b) The rotational resonance frequency vs $R^{-1/2}$. The straight line is the best fit with the slope 1.27. If the abscissa is replaced by $(g/R)^{1/2}$, the slope becomes 0.41.

lyzing the experimental data, we find that the resonance frequency of the rotational mode is proportional to $R^{-1/2}$. On the other hand, the longitudinal resonance frequency is seen to be proportional to $R^{-3/2}$. Figure 5(b) confirms the newly found tendency of the rotational mode, where the resonance frequencies for different liquids collapse into a straight line when plotted versus $R^{-1/2}$. This suggests that the rotational-mode vibration is independent of liquid properties and thus does not follow the physics of surface-tension-controlled vibration, whose angular frequency ω is given by balancing the pressure due to inertial $(\sim\rho R^2\omega^2)$ and capillary force $(\sim\sigma/R)$, where ρ and σ are the density and surface tension, respectively: $\omega\sim(\sigma/\rho R^3)^{1/2}$. Images of Fig. 4(b) confirm that the surface area change during the rotation, thus the role of surface tension force is insignificant. Then the remaining candidate for restoring force is gravity. By balancing the centripetal force $(\sim\rho R^3 R\omega^2)$ with the gravitational force $(\sim\rho R^3 g)$, we find $\omega\sim(g/R)^{1/2}$, which is consistent with the experimental results.

The experimentally found relationship between ω and R for the rotational mode is similar to that of the rotating pendulum with the moment arm R. In Fig. 6, the centripetal force $mr\omega^2$ is balanced by the gravity $mg\tan\phi$, where m is the pendulum mass. For small ϕ, we get $\omega=(g/R)^{1/2}$. Another facet of analogy between the pendulum oscillation and the lowest drop oscillation mode is that the resonance frequencies of the lateral vibration and of the rotation are identical in each system.

In this Letter, we have shown that the lowest oscillation mode of a pedant drop is the rotation around the longitudinal axis, whose frequency $\omega\sim(g/R)^{1/2}$. Before concluding, we point out that Ref. 10 reported a lateral vibration of a sessile mercury drop whose frequency is lower than that of the longitudinal vibration mode. However, their frequencies were

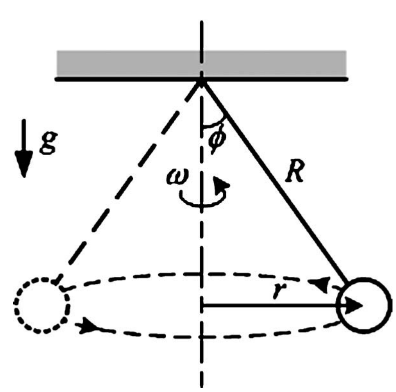

FIG. 6. Pendulum rotating at the angular frequency of ω.

found to follow the law of surface-tension-controlled oscillation, i.e., $\omega \sim (\sigma/\rho R^3)^{1/2}$, which tends to be higher than the frequency of the rotational mode unless R reaches several millimeters. They regarded the rocking vibration as a special case of the standing capillary wave that is qualitatively different from the wave reported here. In the sloshing dynamics, it has been known for a long time that the gravity wave has a lower frequency than capillary wave when the characteristic length is of the order of millimeters.[11] However, our work first reveals the gravity-controlled oscillation of a pendant drop, which has been neither quantitatively measured nor analytically computed to date. Considering that most of previous studies on pendant drops were guided by an assumption that the longitudinal vibration was the most fundamental oscillation mode, a newly found rotational mode is hoped to promote refreshed approaches to this problem. Especially, the low resonance frequency of drop swaying can be applied to microfluidic actuation that uses the lateral vibration of the solid surface.[12]

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