Drop fall-off from the vibrating ceiling

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This work investigates a scheme to rapidly remove liquid drops from a ceiling by vibrating the solid surface. We show that there exist resonant frequencies at which the minimum vibration amplitudes inducing a fall-off of the pendant drops are remarkably less than the detaching amplitudes at neighboring frequencies. These frequencies are shown to correspond to the resonance frequencies of different drop oscillation modes. © *2004 American Institute of Physics.*
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Condensation of vapor into liquid is frequently utilized and observed in industrial situations including condensers of power plants and HVAC (heating, ventilation, and air conditioning) systems. In most condensation processes, vapor changes into liquid drops on a solid surface which is cooler than the saturation temperature. Once formed, the liquid drop reduces the area where the solid and vapor directly exchange heat, and also acts as a thermal resistance between them.[1] Therefore, how to effectively remove such liquid drops from a solid surface exerts a critical influence on the heat transfer rate. There exist many other similar situations where the removal of liquid drops from a solid surface is important. One such example is found in a novel manufacturing process in which molten metal droplets are deposited to form a functional part.[2] In this droplet-based manufacturing process, a high surface tension of molten metal often causes droplets to stick to a droplet generator nozzle, resulting in the nozzle clogging or a low process yield.

Under a gravitational field, a liquid drop can adhere to a ceiling surface due to its surface tension. The corresponding force balance is written as $mg = \pi d\sigma \sin\theta$, where m is the mass of the drop, g is the gravitational acceleration, d is the diameter of the contact area, σ is the surface tension, and θ is the contact angle. Unless an external disturbance is imposed on the pendant drop, it should become heavy enough to overcome the surface force before falling off the ceiling. The simplest form of the external disturbance is perhaps an impulsive motion of the solid surface. However, as everyday experience tells us, it is extremely difficult to make a drop fall off a solid surface by such impulse. Here we show that rather than a great strength of impulsive forcing, a mild vibration of solid surface with an adequate frequency can easily disengage such pendant drops.

A schematic of an experimental apparatus to verify the foregoing argument is shown in Fig. 1. A drop is pendant from a smooth solid surface (Parafilm M Laboratory Sealing Film) attached to a woofer that is vibrated along the direction of gravity. The woofer is driven by an amplifier connected to a function generator. Distilled water is used as a drop liquid and the volume of each drop is kept at $8 \text{ mm}^3 \pm 10\%$, its

equivalent radius of the same volume of sphere being $1.24 \text{ mm} \pm 2.2\%$.

To measure drop oscillation and the woofer's vibration amplitude, a high speed video system (Kodak Ektapro EM) is used at a rate of 1000 frames per second. The following experimental procedure was performed to obtain the minimum forcing magnitude (vibration amplitude, velocity, or acceleration) that induces the disengagement of a liquid drop from a solid surface as a function of frequency: (1) A drop is placed on a fresh Parafilm surface facing downward. (2) The frequency of a sinusoidal wave, f, from a function generator is adjusted. (3) The amplifier output voltage is increased from zero to increase the woofer vibration. (4) Drop oscillation gets vigorous with the increase of woofer vibration magnitude, until the drop falls off the solid surface. The high speed video system is triggered by the drop and images of drop disengagement are recorded. (5) The foregoing procedure is repeated at a different frequency with new drop and surface.

By analyzing the experimental images, parameters such as drop length (L), maximum diameter (D), contact area diameter (d), and woofer vibration amplitude (A) are measured for each frame (see Fig. 2). The woofer vibration measured based on the images at the moment of drop disengagement corresponds to the minimum forcing that is able to induce drop fall-off. The representative magnitudes of the vibration velocity, U, and the acceleration, a, are respectively obtained as $U = 2\pi f A$ and $a = (2\pi f)^2 A$ since the plate motion is sinusoidal.

The experimentally obtained minimum forcing magnitudes that can induce drop fall-off as a function of frequency are shown in Fig. 3. Circles and error bars respectively correspond to the average and standard deviation of at least three experimental data for each frequency. The line represents a fourth order polynomial determined by the least square method based on the experimental data. The figures show that there exist two local minima in all the plots for the amplitude, the velocity, and the acceleration. That is, in specific frequency ranges around 25 Hz and 75 Hz a drop easily disengages from a solid surface with a weak external vibration, whereas a relatively strong vibration is necessary to induce drop fall-off in other frequency ranges including a case of impulsive forcing.

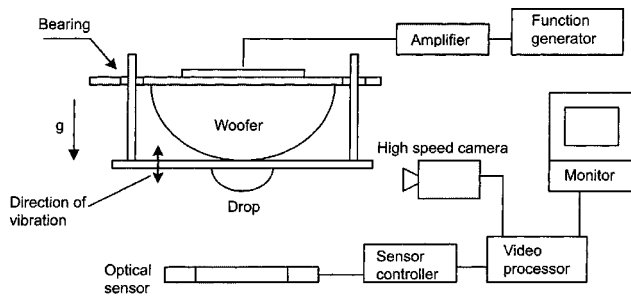[a]Electronic mail: hoyoung@kist.re.kr

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FIG. 1. Experimental setup.

We relate these observations to the resonance of drop oscillation which takes place when the drop resonant frequency and forcing frequency coincide. Figure 4 shows the high speed images of drops separating from a surface at external forcing frequencies of 25 Hz and 75 Hz. At a low frequency region, such as in Fig. 4(a), the drop oscillates between prolate and oblate shapes, behaving similar to the second mode shape of a freely vibrating drop. On the other hand, at a high frequency region, such as in Fig. 4(b), the drop shape is more complex and rather resembles that of the third mode oscillation of a freely vibrating drop. Therefore, we conclude that the shape oscillation mode of a pendant drop changes according to the external forcing frequency as that of a freely vibrating drop. This implies that there exist
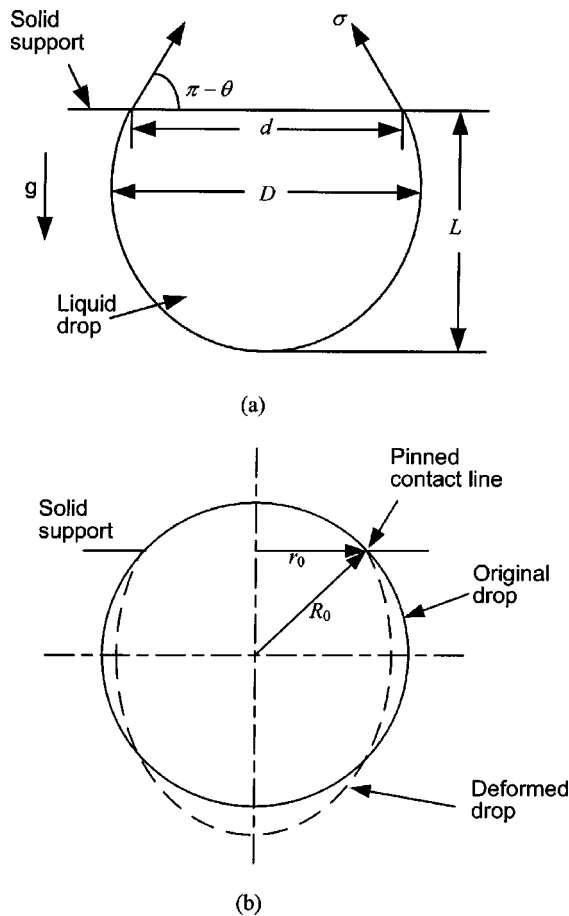

FIG. 2. Configurations of a liquid drop pendant from (a) a flat surface and (b) a spherical bowl.
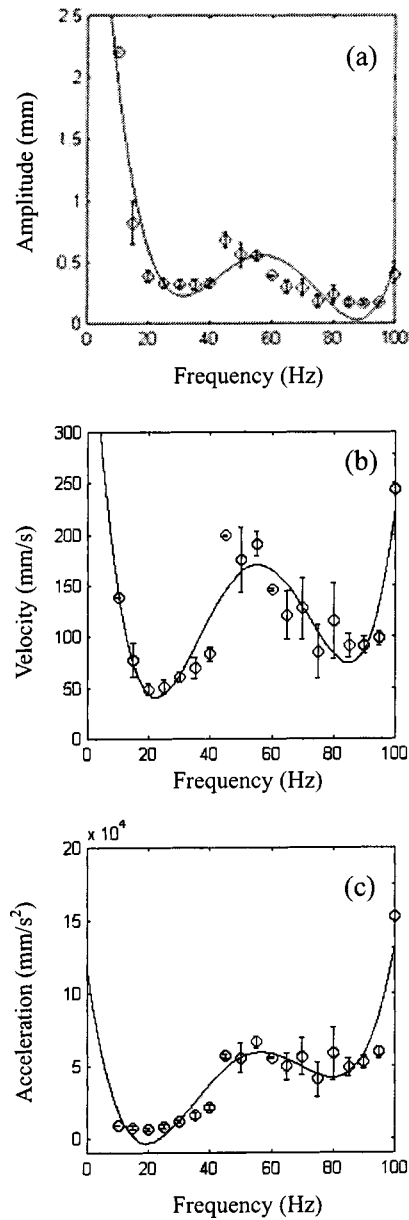

FIG. 3. Minimum forcing magnitudes which induce drop disengagement at each frequency. (a) Amplitude, (b) velocity, (c) acceleration.

more than one resonant frequencies corresponding to each oscillation mode, which is consistent with the current experimental observations.

To find the fundamental oscillation characteristics of a pendant drop, the dynamic response of a drop to impulsive forcing has been measured using a high speed video system. The impulse is generated by abruptly stopping a plate which is being pulled upward. Temporal evolutions of d, D, and L are measured and the fast Fourier transform (FFT) analysis is conducted to find the most easily excitable frequency. The results show that the dominant frequency of the drop response is found around 25 Hz corresponding to the first local minima in Fig. 3. However, the higher-order mode oscillations were not accessible via a current impulsive method. This limitation was also reported for pendant drops whose contact lines were fixed.[3]

Figure 5 provides a phenomenological explanation to the

Initial state $t = -53$ ms $t = -45$ ms $t = -34$ ms

$t = -25$ ms $t = -14$ ms $t = -2$ ms $t = 0$

(a)

$\lfloor$____ 3 mm ____$\rfloor$

Initial state $t = -58$ ms $t = -54$ ms $t = -50$ ms

$t = -19$ ms $t = -14$ ms $t = -4$ ms $t = 0$
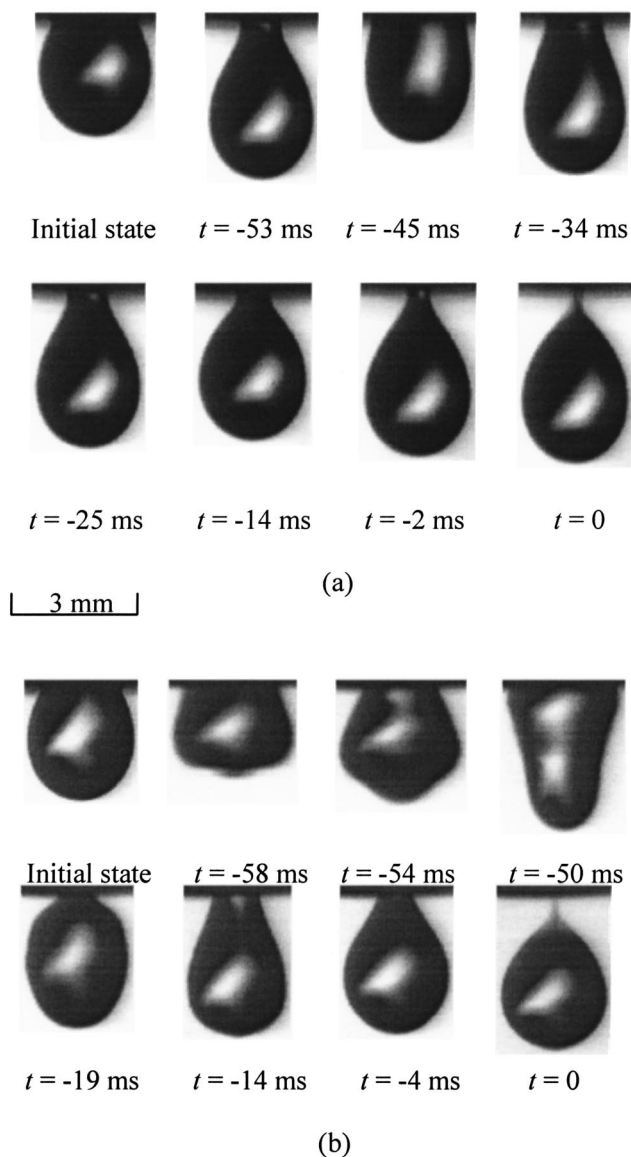
(b)

FIG. 4. High speed images of the drops disengaging from a vibrating ceiling. The value of t for each image indicates a time relative to the moment ($t = 0$) the drop separates from a surface. The forcing frequencies are (a) 25 Hz and (b) 75 Hz.

FIG. 5. Evolution of the contact diameter d prior to drop disengagement. (a) $f = 25$ Hz, (b) $f = 50$ Hz, (c) $f = 75$ Hz, (d) $f = 100$ Hz.

easy disengagement of a drop at specific frequencies. This figure exhibits the temporal evolution of the drop contact diameter prior to drop fall-off. At frequencies of 25 Hz and 75 Hz, where a drop easily disengages, d repeats its increase and decrease synchronously with the external forcing. On the other hand, d shows inharmonious behavior at other frequencies indicating that the drop motion and external forcing are discordant. Therefore, we again find that the external forcing frequency should accord with the pendant drop's shape oscillation to achieve an effective disengagement of the drop.

According to the current experimental measurements, the resonant frequencies related to disengagement of the pendant drop used in this work are found to be about 25 Hz and 75 Hz for the lowest and the second lowest mode oscillations, respectively. We compare these frequencies with values obtained by idealized models. The most easily accessible
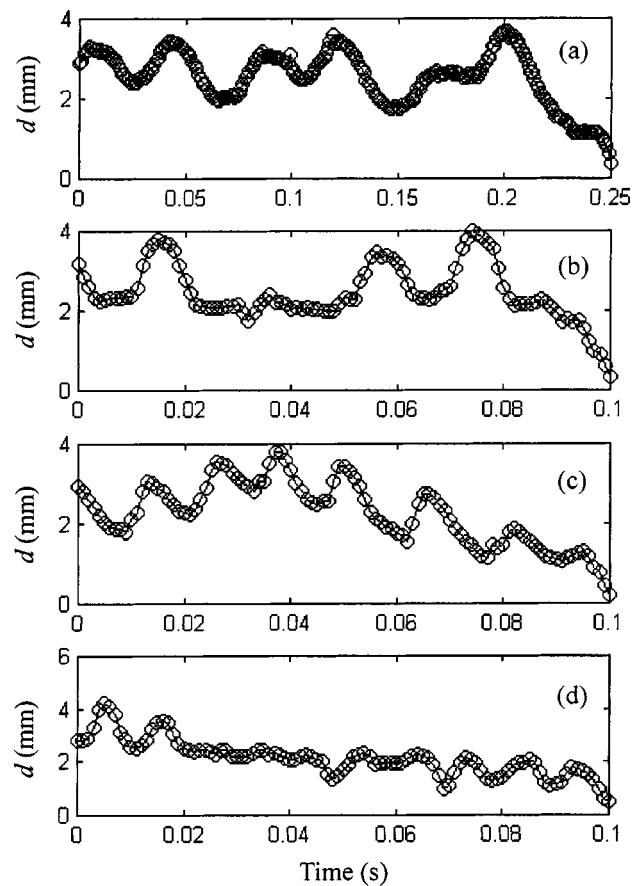
theory relevant to this problem is that for the vibration of a free drop.[4] The natural frequency of the nth mode vibration of an inviscid drop, f_n, is given by

$$f_n = \frac{1}{2\pi}\left[n(n-1)(n+2)\frac{\sigma}{\rho R^3}\right]^{1/2}, \tag{1}$$

where σ, ρ, and R are the surface tension, the density, and the radius of the drop, respectively. The mode number n denotes the number of nodes occurring in the oscillation. The second and the third mode oscillations are similar to the drop shapes shown in Figs. 4(a) and 4(b), respectively. Since the drop under current investigation is in contact with a solid surface, different lengths can be chosen to replace R in Eq. (1), resulting in different frequency values. Here we select three length values, that is, the radius of a spherical drop having the identical volume as the current drop (model I), $D/2$ (model II), and L (model III).

Besides the foregoing classical theory of free drop vibration, Strani and Sabetta[5] developed a theory to predict the natural frequency of a small inviscid drop in partial contact with a spherical bowl, using the Green function method combined with the Legendre series expansion. The drop configuration analyzed by Ref. 5 is shown in Fig. 2(b). Such constrained drop was found to have an additional, first vibration mode, whose shape is close to that of the free drop's second mode oscillation. Although developed for the solid support shaped like a spherical bowl, the theory is reported to cor-

TABLE I. Theoretically predicted resonance frequencies.

Model	f_l (Hz)	f_h (Hz)
I	87	169
II	86	167
III	27	53
IV	51	127
V	36	88

rectly predict the natural frequency of nonwetting drops sitting on a flat surface with fixed contact lines as well.[6] The natural frequency of such drops is determined by the drop base radius r_0 (or the radius of bowl opening) and the drop radius R_0. The drop base radius is measured straightforward from the experimental image but different values for R_0 can be chosen. When the gravitational effect is negligible, the drop should be a spherical cap having a constant radius of curvature. However, the drop used in this study elongates in the gravitational direction in spite of its radius being smaller than the capillary length of water (2.7 mm). One obvious choice is $R_0 = D/2$ (model IV). An alternative value is obtained as $R_0 = L/(1 - \cos\theta)$ which is exact for a spherical cap (model V).

Table I lists theoretical values determined by each model. In the table, f_l and f_h denote the lower-mode and higher-mode frequency, respectively, with $l = 2$ and $h = 3$ for models I, II, and III, and $l = 1$ and $h = 2$ for models IV and V. Model V yields the closest predictions to the experimentally obtained frequencies in both modes. It is remarkable that the model of Ref. 5 closely predicts frequencies of vibration that easily disengage pendant drops in spite of different conditions of the contact line mobility and large oscillation amplitude beyond linear regime. We also note that the lower-mode frequency predicted by model III is close to the experimentally determined value although the higher-mode frequency shows a great discrepancy. It can be readily shown that f_l values based on models III and V are inherently very similar for pendant drops with the contact angle around 115°, which is the case here.

Here we note that numerical models are available for the oscillation of drops pendant from a flat solid surface.[7,8] Although Wilkes and Basaran[8] modeled based on the fixed contact line, they obtained distinct resonance forcing frequencies and found the degree of oscillation at higher resonance frequencies to be weaker than that at the first resonance mode.

Both of the foregoing observations accord with the current experimental results. Since the analytical model assuming pinned contact line predicts the resonant frequency of pendant drops involving moving contact line reasonably well, using the foregoing numerical model is expected to give more accurate predictions than the analytical models.

In summary, we have shown that a liquid drop pendant from a solid surface can be effectively detached by vibrating the solid with adequate frequencies. The drop oscillation at those frequencies exhibits resonant behavior with the solid motion. The theory of Ref. 5 is shown to closely predict such resonant frequencies although the experimental conditions deviate from idealized assumptions. Although not covered in this work, higher frequencies than 100 Hz will result in further local minima in forcing strength to induce drop fall-off. However, the practicality of using that frequency range is low considering a high energy input to generate vibrations of such high frequency.

One example of utilizing this scheme for drop production is detaching a molten metal drop produced by laser or electric discharge arc. The molten drops, used in droplet welding or free form fabrication, can easily fall off by external vibration at the drop's resonant frequency. In this way, the drop production rate can be enhanced as compared with the conventional processes that do not use external vibrations. Based on the observations made in this work, we note that various methods to impose vibration on a drop can be devised for industrial applications. In addition to the solid plate vibration, alternating electric field or periodic acoustic pressure can also be considered feasible candidates for this scheme.

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